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Supersymmetric dissipative quantum mechanics from superstrings

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Abstract: Following the approach of Callan and Thorlacius applied to the superstring, we derive a supersymmetric extension of the non-local dissipative action of Caldeira and Leggett. The dissipative term turns out to be invariant under a group of superconformal transformations. When added to the usual kinetic term, it provides an example of supersymmetric dissipative quantum mechanics.

Keywords: Superstrings and Heterotic Strings, Supersymmetric Effective Theories, Quantum Dissipative Systems.

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Contents

1. Introduction

Dissipative classical and quantum systems have constantly received attention, along the years, by both theoreticians and experimentalists. On one side, this subject is strictly related to other interesting questions about decoherence, and non-equilibrium physics of open systems with a small number of degrees of freedom interacting with an external environment. On the other side, it could be relevant in explaining experimental results in the physics of mesoscopic systems.

A model describing the dissipative behavior of a quantum mechanical system was given long ago by Caldeira and Leggett (CL) [1]. Briefly speaking, the CL model of dissipative quantum mechanics consists in considering a quantum system in interaction with a bath formed by an infinite number of harmonic oscillators. If the interaction is linear and if the spectral function of the frequencies of the oscillators satisfies what is called the Ohmic condition, the bath produces a non-local effective term which introduces dissipation in the dynamics of the quantum system. Some time later, Callan and Thorlacius [2] found an interesting derivation of the CL model in string theory. They showed that open bosonic string theory provides a way of generating the CL non-local term as the effective action describing the interaction of the string end-point with the oscillation modes of the string. The original derivation of [2] follows methods developed in [3] and [4], and uses the formalism of boundary states [5, 6]. A somewhat simpler derivation of their result, making use of a (constrained) functional integration was given in [7].

In this paper we show that similar reasonings, applied to the fermionic string, produce a new non-local effective action which is the fermionic analogous of the CL non-local term. When contributions from the bosonic and fermionic coordinates of the string are considered, one gets both the CL term and its fermionic companion. We find that they are separately invariant under $\mathrm{SL}(2,\mathbb{R})$ subgroup of the full reparametrization group of the string, and that when they are put together the resulting expression is supersymmetric. It can be explicitly written in superspace formalism and turns out to be invariant under superconformal transformations. When it is added to a supersymmetric kinetic term, it provides the first example of a supersymmetric dissipative quantum mechanics, which is the main result of this paper.

The plan of the paper is as follows. In section 2 we give a short review of the Caldeira-Leggett construction of the non-local action which describes dissipation in a quantum mechanical system. In section 3 derive the analogous non-local action for a fermionic system, following the method of constrained functional integration, of ref. [7]. In section 4, another derivation, which is closer in spirit to the original derivation of [2] is given using the boundary state formalism. In section 5, we show that the non-local fermionic dissipative term is invariant under the $\mathrm{SL}(2,\mathbb{R})$ subgroup of the full reparametrization group of the string. Furthermore, we show that the bosonic (CL) and fermionic terms are parts of a superconformal invariant action which we write it explicitly in superspace. In section 6, the dissipative non-local term is added to a supersymmetric kinetic term to form the action of a supersymmetric dissipative quantum mechanical system. Some further discussions can be found in the conclusive section.

2. The Caldeira-Leggett model of dissipative quantum mechanical systems

The simplest example of classical dissipative system is the particle moving in the presence of friction. Its equation of motion is

$$M\frac{d^2q}{dt^2}+\eta\frac{dq}{dt}+\frac{dV_0(q)}{dq}=0$$

where $V_0(q)$ is the potential and η gives the strength of the friction force. As is well known there is no action whose variation can produce an equation of motion containing the term proportional to η, and thus there is no hamiltonian structure and no canonical quantization procedure for this system.

According to Caldeira and Leggett [1], quantum dissipative mechanical behavior can be obtained from a microscopic model consisting in coupling the mesoscopic system with an environment modelled as an infinite set of harmonic oscillators. The CL microscopic action of the particle on the line can then be written as follows:

$$\begin{aligned} S[q,\{x_\alpha\}] = \int dt \Bigg[& \frac{1}{2}M\left(\frac{dq}{dt}\right)^2 - V_0(q) + \sum_\alpha \left(\frac{1}{2}m_\alpha\left(\frac{dx_\alpha}{dt}\right)^2 - \frac{1}{2}m_\alpha\omega_\alpha^2 x_\alpha^2\right) - \\ & - q\left(\sum_\alpha C_\alpha x_\alpha + F_{ext}(t)\right) - \sum_\alpha \kappa\frac{C_\alpha^2 q^2}{2m_\alpha\omega_\alpha^2}\Bigg], \end{aligned} \tag{2.1}$$

where $q(t)$ is the dynamical variable of the mesoscopic system and $\{x_\alpha(t)\}$ is the infinite set of harmonic oscillators representing the environment. Without lack of generality one can consider that the system is coupled linearly to each oscillator with strength C_α. $F_{ext}(t)$ represents an external force. The last term in (2.1) has to be introduced to take into account a renormalization effect to the classical potential $V_0(q)$. The classical equations of motion of (2.1) are

$$\begin{aligned} M\frac{d^2q}{dt^2} &= -\frac{dV_0(q)}{dq} - \sum_\alpha \left(C_\alpha x_\alpha + \kappa \frac{C_\alpha^2 q}{m_\alpha \omega_\alpha^2} \right) + F_{ext}(t) \\ m_\alpha \frac{d^2 x_\alpha}{dt^2} &= -m_\alpha \omega_\alpha^2 x_\alpha - C_\alpha q \end{aligned} \tag{2.2}$$

Fourier transforming, solving for x_α in the second equation and plugging into the first equation, one obtains

$$-M\omega^2 \widetilde{q}(\omega) = -\widetilde{\frac{dV_0}{dq}}(\omega) + \widetilde{F}_{ext}(\omega) + \widetilde{K}(\omega)\widetilde{q}(\omega) \tag{2.3}$$

where

$$\widetilde{K}(\omega) = -\sum_\alpha \frac{C_\alpha^2}{m_\alpha \omega_\alpha^2} \frac{\omega^2}{\omega^2 - \omega_\alpha^2} \, .$$

One gets an imaginary part $\widetilde{J}(\omega) = \operatorname{Im} \widetilde{K}(\omega)$ by the usual rule $\omega \to \omega + i\varepsilon$, which shifts the poles from the real axis:

$$\widetilde{J}(\omega) = \frac{\pi}{2} \sum_\alpha \frac{C_\alpha^2}{m_\alpha \omega_\alpha^2} \delta(\omega - \omega_\alpha) \, .$$

(Notice that expression of $\widetilde{J}(\omega) = \operatorname{Im} \widetilde{K}(\omega)$, is independent of the presence of the last term in (2.1), which only affects the real part of $\widetilde{K}(\omega)$. On the other hand, $\widetilde{K}(\omega)$ is related to $\widetilde{J}(\omega)$ by a subtracted dispersion relation when the renormalization correction term is taken into account. One can then show that in that case $\widetilde{J}(\omega) = \operatorname{Im} \widetilde{K}(\omega)$ is the dominant contribution with respect to $\operatorname{Re} \widetilde{K}(\omega)$ for ω below some characteristic frequency of the system.) If the parameters of the bath of harmonic oscillators satisfy the Ohmic condition

$$\widetilde{J}(\omega) = \eta\omega, \tag{2.4}$$

then the eq. (2.3) reduces to

$$-M\omega^2 \widetilde{q}(\omega) = -\widetilde{\frac{dV_0}{dq}}(\omega) + \widetilde{F}_{ext}(\omega) + i\eta\omega\widetilde{q}(\omega) \tag{2.5}$$

i.e. an effective friction term is generated in the equation of motion of the particle.

Quantum-mechanically, the interaction of the particle with the oscillator bath produces a non-local effective action term, once the euclidean functional integration on the $x_\alpha(t)$ is performed. The result is

$$S_{CL}[q] = -\int dt_1 \int dt_2 q(t_1)\alpha(t_1 - t_2)q(t_2),$$

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where the non local coupling coefficient is given by

$$\alpha(t) = \int_0^\infty \frac{d\omega}{2\pi} \widetilde{J}(\omega) e^{-\omega|t|}.$$

In case of Ohmic dissipation (2.4), one obtains the Caldeira-Leggett non local term [1]

$$S_{CL}[q] = \frac{\eta}{2\pi} \int dt_1 \int dt_2 \frac{q(t_1)q(t_2)}{(t_1 - t_2)^2} = \frac{\eta}{4\pi} \int dt_1 \int dt_2 \frac{(q(t_1) - q(t_2))^2}{(t_1 - t_2)^2}, \tag{2.6}$$

The introduction of the CL term has been seminal in the study of dissipative quantum mechanics. A general reference on the subject is [8].

Working in two-dimensional bosonic string theory, Callan and Thorlacius [2] found a connection between the so-called boundary states and one-dimensional dissipative quantum mechanics. Roughly speaking, boundary states are a sort of coherent states in the Fock space of string theory, which implement operatorially open string boundary conditions. (On this point we shall be much more explicit in section 4.) The main idea is that, while evolving in time, the open string end-point behaves like a point particle interacting with the infinite set of harmonic oscillator formed by the string modes. As such, its dynamics is dissipative. In the next section we illustrate how a dissipative non-local action for the dynamics of the string end-point, can be obtained by functional-integrating out the open string oscillator modes.

3. A dissipative term from the fermionic string

Our first derivation of the fermionic non-local dissipative term follows the approach used in ref. [7] to derive the CL lagrangian from the open bosonic string. It consists in the direct evaluation of a constrained open string partition function, where the string fields are bound to assume some fixed boundary values at one of the string end-points.

Let us begin by writing the action of the fermionic string as [9]

$$\mathcal{S} = -\frac{1}{4\pi\alpha'} \int_{-T}^{T} d\tau \int_0^\pi d\sigma \left(\partial_a X_\mu \partial^a X^\mu - i\overline{\psi}_\mu \rho_a \partial^a \psi^\mu \right). \tag{3.1}$$

Here the $X = X^\mu(\tau, \sigma)$ are the bosonic coordinates of the string, the $\psi = \psi^\mu(\tau, \sigma)$, are massless Majorana spinors on the string world-sheet, and the ρ_a, $a = 0, 1$ are two-dimensional Dirac matrices. In the following, we shall momentarily neglect the bosonic part of the action, and consider only a single fermionic field. Using light-cone coordinates, the fermionic action can be written

$$\mathcal{S} = \frac{i}{4\pi\alpha'} \int d\tau d\sigma \left(\psi_- \partial_+ \psi_- + \psi_+ \partial_- \psi_+ \right).$$

Boundary conditions can be written in terms of $\psi_-(\tau, \sigma)$ defined on the doubled interval $\in (0, 2\pi)$, with $\psi_+(\tau, \sigma) = \psi_-(\tau, 2\pi - \sigma)$. The Ramond and Neveu-Schwartz sectors correspond to $\psi_-(\tau, \sigma)$ periodic or anti-periodic on the doubled interval. The corresponding mode expansions are of the form

$$\psi_\pm(\tau, \sigma) = \sum_{p=-\infty}^{\infty} \psi_p(\tau) e^{\pm ip\sigma}, \tag{3.2}$$

where p is integer in the Ramond sector and half-integer in the Neveu-Schwartz case. We then perform a Wick rotation in time $\tau \to it$ and Fourier expand also in $t \in (-T, T)$, obtaining

$$\psi_{\pm}(t,\sigma) = \sum_{p=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \psi_{p,k} e^{i(\frac{2\pi k}{T}t \pm p\sigma)}$$

with $(\psi_{p,k})^* = \psi_{-p,-k}$, following from the reality of $\psi_{\pm}(t,\sigma)$. Substituting these expansions into the euclidean action, we obtain after some straightforward algebra

$$S_E = \frac{T}{\alpha'} \sum_{p=-\infty}^{\infty} \sum_{k=0}^{\infty} (p + \frac{2\pi i}{T} k) \psi_{-p,-k} \psi_{p,k} \,.$$

Consider now the mode expansion of the assigned boundary value of the fermionic field, i.e.,

$$\psi_{\pm}(t,0) = \sum_{p=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \psi_{p,k} e^{\pm i(\frac{2\pi i k}{T}t)} = \sum_{k=-\infty}^{\infty} \chi_k e^{\pm i(\frac{2\pi i k}{T}t)} \equiv \chi(t) \,.$$

The one-dimensional effective action for the string end-point fermionic variable $\chi(t)$, is obtained by computing the following constrained functional integral, which is the annulus diagram of the open string, with the assigned boundary conditions

$$e^{-S_{\text{eff}}(\chi)} = \mathcal{N} \int \left[\prod_{k>0} \prod_{p} d\psi_{-p,-k} d\psi_{p,k} \right] \prod_{k} d\psi_{0,k} \, e^{-\mathcal{S}_E} \prod_{k} \delta \left(\sum_{p} \psi_{pk} - \chi_k \right) ,$$

The constraints, given by delta functions, are easily solved, giving $\psi_{0,k} = \chi_k - \sum_{p\neq 0} \psi_{p,k}$. The resulting integrals for each k are then of the form (we momentarily drop the factor T/α' from $\mathcal{S}_E$)

$$\begin{aligned} e^{-S(\chi_k)} = \mathcal{N} e^{\frac{2\pi i k}{T} \chi_{-k}\chi_k} & \int \prod_{p} d\psi_{-p,-k} d\psi_{p,k} \times \\ & \times \exp \left\{ \frac{2\pi i k}{T} \left[\chi_{-k} \left(\sum_{p\neq 0} \psi_{p,k} \right) + \left(\sum_{p\neq 0} \psi_{-p,-k} \right) \chi_k \right] + \right. \\ & \left. + \sum_{p'\neq 0} \sum_{p\neq 0} \psi_{-p',-k} \left[\frac{2\pi i k}{T} + \delta_{pp'} (p + \frac{2\pi i k}{T}) \right] \psi_{p,k} \right\} , \end{aligned} \tag{3.3}$$

All integrals in (3.3) are gaussian and can be done by completing the squares. Eventually, for each mode χ_k one obtains

$$S(\chi_k) = \frac{1}{\sum_{p=0} 1/a(p)} \,, \tag{3.4}$$

with

$$a(p) = \left(p + \frac{2\pi i k}{T} \right) . \tag{3.5}$$

The sum (3.4) is not converging (but in a distributional sense) for $a(p)$ given in (3.5), nevertheless, one could regularize it by adding, for instance, a term εp^2 to $a(p)$, which would make it converging, and then passing to the $\varepsilon \to 0$ limit after having summed (using residues). One obtains

$$S(\chi_k) = \frac{i}{\pi} \tanh\left(\frac{2\pi^2 k}{T}\right) \chi_{-k}\chi_k \tag{3.6}$$

Alternatively, referring to [7] for the details, one can introduce some dimensional parameter into the action to define a continuum limit, which corresponds to the case where the inner circular border of the annulus shrinks to zero, i.e. to the disk diagram of the string, and replace the sum over p by an integral, to get

$$S(\chi_k) = \frac{1}{\int_0^\infty dx/\left(\frac{2\pi i k}{T} + x\right)} \chi_{-k}\chi_k = \frac{i}{\pi}\,\mathrm{sign}(k)\,\chi_{-k}\chi_k\,. \tag{3.7}$$

where the integral has to be considered as principal value. It easily seen that the two expressions (3.6) and (3.7) tend to coincide for high values of k, i.e. at high frequencies, where the continuum limit is obviously a good approximation of the discrete sum, while they differ at low frequencies.

We get the effective action on the circle of length $2T$, corresponding to finite temperature, by summing on k, the various contributions in (3.7). However, when dealing with fermions we can consider periodicity or anti-periodicity in euclidean time too, corresponding to integer or half-integer values of k. Using string world-sheet duality, which exchanges the open and closed string channel, (anti-)periodicity in time of the open string fermions will correspond to closed string fermions (anti-)periodic in the string coordinate σ.

The resulting action for the theory on the circle, restoring the factor T/α', takes the two different non-local expressions:

$$S(\chi) = \frac{1}{4\pi\alpha'}\frac{1}{T}\mathrm{P}\int_{-T}^{T} dt \int_{-T}^{T} dt' \chi(t)\chi(t') \cot\left[\frac{\pi(t-t')}{T}\right], \tag{3.8}$$

for fermions periodic in euclidean time, and

$$S(\chi) = \frac{1}{4\pi\alpha'}\frac{1}{T}\mathrm{P}\int_{-T}^{T} dt \int_{-T}^{T} dt' \frac{\chi(t)\chi(t')}{\sin[\pi(t-t')/T]}, \tag{3.9}$$

for antiperiodic ones. The symbol P indicates that the integral is principal value.

Both expressions (3.8) and (3.9) lead to the same non local action in the zero temperature $T \to \infty$, i.e.

$$S(\chi) = \frac{1}{4\pi^2\alpha'}\mathrm{P}\int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \frac{\chi(t)\chi(t')}{(t-t')}\,. \tag{3.10}$$

This term is the analogue, for the anticommuting variable $\chi(t)$ of the CL dissipative term for the commuting variable $q(t)$.

For the sake of comparison, we summarize here how the standard CL action was obtained in ref. [7] starting from the bosonic term of the string action. The expression of $a(p)$ is different from (3.5), because the bosonic term is quadratic in the derivatives and one gets

$$a(p) = \left(p^2 + \left(\frac{2\pi k}{T}\right)^2\right).$$

The corresponding sum in (3.4) is now converging and using residues one obtains

$$S(q_k) = \frac{2\left(\frac{2\pi k}{T}\right)^2}{1+\pi\left(\frac{2\pi k}{T}\right)\coth(\frac{2\pi^2 k}{T})} q_{-k} q_k \tag{3.11}$$

where the q_k are the Fourier modes of the boundary conditions of the bosonic field at $\sigma = 0$. In the continuum limit the sum can be replaced by an integral and one gets [7]

$$S(q_k) = -\frac{T}{4\pi\alpha'} \frac{1}{\int_0^\infty dx / \left(\left(\frac{2\pi k}{T}\right)^2 + x^2\right)} q_{-k} q_k = -\frac{1}{\alpha'} |k|\, q_{-k} q_k \,. \tag{3.12}$$

Summing over all the integer values k, one obtains the CL non local action on the circle

$$S(q) = -\frac{1}{16\alpha'} \frac{1}{T^2} \mathrm{P} \int_{-T}^{T} dt \int_{-T}^{T} dt' \frac{q(t) q(t')}{(\sin[\pi(t-t')/T])^2}, \tag{3.13}$$

which in the limit $T \to \infty$, reduces to the CL dissipative term

$$S(q) = -\frac{1}{16\pi^2\alpha'} \mathrm{P} \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \frac{q(t) q(t')}{(t-t')^2}, \tag{3.14}$$

with dissipation constant η proportional to the inverse of α'.

4. Derivation using boundary states

The original derivation of the Caldeira-Leggett non-local action in string theory was done in [2] using the formalism of boundary states. It is then not surprising that the fermionic non local action can be analogously obtained using fermionic string boundary states. Let us first summarize the method in the bosonic case [4]. (In this section we take $\alpha' = 2$, restoring it only in the final formulas.) A single closed bosonic string coordinate can be expanded in modes as follows

$$X(\tau, \sigma) = q - 2ip\tau + \sum_{m \neq 0} \frac{1}{\sqrt{|m|}} \left(a_m e^{-m(\sigma + i\tau)} + \tilde{a}_m e^{-m(\sigma - i\tau)} \right),$$

where the only non vanishing commutation relations of the left and right moving creation and annihilation are $\left[a_m, a_n^\dagger\right] = \left[\tilde{a}_m, \tilde{a}_n^\dagger\right] = \delta_{mn}$, and $a_n^\dagger = a_{-n}$, $\tilde{a}_n^\dagger = \tilde{a}_{-n}$ The boundary state $|x, \overline{x}\rangle$ is a coherent state satisfying the equations

$$\begin{aligned} (a_m^\dagger + \tilde{a}_m - x_m)|x, \overline{x}\rangle &= 0\,, \\ (\tilde{a}_m^\dagger + a_m - \overline{x}_m)|x, \overline{x}\rangle &= 0\,, \end{aligned} \qquad m > 0$$

and the completeness condition

$$\int \mathcal{D}x\, \mathcal{D}\overline{x}\, |x, \overline{x}\rangle \langle x, \overline{x}| = 1$$

The solution is

$$|x, \overline{x}\rangle = \exp\left[-\left(a^\dagger, \tilde{a}^\dagger\right) - \frac{1}{2}(\overline{x}, x) + \left(a^\dagger, x\right) + \left(\overline{x}, \tilde{a}^\dagger\right) \right] |0\rangle, \tag{4.1}$$

where the scalar product is defined as $(a,b) = \sum_{m>0} a_m b_m$. The general bosonic boundary state can be written

$$|B_X\rangle = \int \mathcal{D}x\, \mathcal{D}\overline{x}\, e^{-S(x,\overline{x})}|x,\overline{x}\rangle\,,$$

where $S(x,\overline{x})$ is a bosonic boundary action, describing the interaction of the open strings with background fields. When the latter is zero one recovers the usual Neumann boundary state

$$|N_X\rangle = \int \mathcal{D}x\, \mathcal{D}\overline{x}\, |x,\overline{x}\rangle = \exp\left(a^\dagger, \widetilde{a}^\dagger\right)|0\rangle\,.$$

According to Callan and Thorlacius [2], terms depending on x and $\overline{x}$ in the exponential in (4.1) can also be interpreted as the action of a quantum mechanical variable defined at the string end point and interacting with external sources given by the right and left moving creation operators. (Notice that these last all commute with each other and can be considered as c-number sources. They were actually used as external sources in order to derive reparametrization invariance Ward identities [10], searching for conformal points, i.e. phase transitions, of dissipative models [11]. Early attempts to generalize the above mentioned reparametrization Ward identities to the supersymmetric case can be found in [12].) The term, quadratic in x and $\overline{x}$, is just the CL term on the circle

$$\frac{1}{2}\left(\overline{x},x\right) = -\frac{i}{4\pi}\int_0^{2\pi} X_+(\sigma)\frac{dX_-(\sigma)}{d\sigma}d\sigma = -\frac{1}{32\pi^2\alpha'}\int_0^{2\pi} d\sigma \int_0^{2\pi} d\sigma' \frac{X(\sigma)X(\sigma')}{\sin^2\left(\frac{\sigma-\sigma'}{2}\right)}, \tag{4.2}$$

where, in the second identity $X_\pm(\sigma)$ are the positive and negative frequency parts of $X(\sigma)$. In the last identity we have restored α' . This term is the bosonic string action evaluated on the solution of the equation of motion which is regular in the interior of the disk.

It is straightforward to show that, in the case of the fermionic string, the same method leads to a non local fermionic action. To avoid complications due to the zero modes which are present in the Ramond sector, we limit our discussion to Neveu-Schwartz fermions, which are antiperiodic in the variable σ. The fermionic boundary state is defined through the equations

$$\begin{aligned}(\psi_m^\dagger \pm i\widetilde{\psi}_m - \overline{\chi}_m)|\chi,\overline{\chi};\pm\rangle &= 0\,,\\ (\widetilde{\psi}_m^\dagger \pm i\psi_m - i\chi_m)|\chi,\overline{\chi};\pm\rangle &= 0\,,\end{aligned} \qquad m>0 \tag{4.3}$$

and with the correct normalization given by

$$|\chi,\overline{\chi},\pm\rangle = \exp\left[\pm i\left(\psi^\dagger,\widetilde{\psi}^\dagger\right) - i\left(\overline{\chi},\chi\right) + i\left(\psi^\dagger,\chi\right) \pm \left(\overline{\chi},\widetilde{\psi}^\dagger\right)\right]|0\rangle\,. \tag{4.4}$$

Following [13], we have redefined in (4.3) and (4.4) the variable χ and $\overline{\chi}$, with respect to [4], in order to write the amplitude as a classical action. The general fermionic boundary state reads

$$|B_\chi,\pm\rangle = \int \mathcal{D}\chi\,\mathcal{D}\overline{\chi}\, e^{-S(\chi,\overline{\chi})}|\chi,\overline{\chi},\pm\rangle\,,$$

where, as in the bosonic case, $S(\chi,\overline{\chi})$ is a one-dimensional boundary action.

Extending the interpretation of [2] to the fermionic case, terms depending on χ and $\overline{\chi}$ in the exponential in (4.4) can also be considered as the action of a one-dimensional anti-commuting variable defined at the string end point and interacting with external sources.

The fermionic non local term on the circle is given by

$$-i\,(\overline{\chi},\chi)=\frac{1}{8\pi^2\alpha'}\mathrm{P}\int_0^{2\pi}d\sigma\int_0^{2\pi}d\sigma'\,\frac{\chi(\sigma)\chi(\sigma')}{\sin\left(\frac{\sigma-\sigma'}{2}\right)}. \tag{4.5}$$

To compare (4.2) and (4.5) with the results of section 3, one has to make a string world-sheet duality transformation $\sigma\to(2\pi/T)\,t$, remembering that world-sheet fermions contribute an additional factor $(2\pi/T)$ because they have conformal weight $1/2$. One then recovers eqs. (3.13) and (3.9).

5. $\mathrm{SL}(2,\mathbb{R})$ symmetry and supersymmetry

It was noticed in [2] that the non-local Caldeira-Leggett term on the real line is invariant only under the $\mathrm{SL}(2,\mathbb{R})$ (or under $\mathrm{SU}(1,1)$ for the theory on the circle) subgroup of the full infinite group of reparametrizations. Full invariance would be recovered only at critical points, if they exist. One can see that the fermionic action (3.10) is invariant under the same $\mathrm{SL}(2,\mathbb{R})$ group. The key point is that χ has a two-dimensional origin and as such has a conformal dimension $1/2$. Then under a reparametrization $\tau\to\widetilde{\tau}=f(\tau)$ it transforms as $\widetilde{\chi}(\tau)=\left(f'(\tau)\right)^{1/2}\widetilde{\chi}(\widetilde{\tau})$, then

$$\int_{-\infty}^{\infty}d\tau\int_{-\infty}^{\infty}d\tau'\,\frac{\chi(\tau)\chi(\tau')}{\tau-\tau'}=\int_{-\infty}^{\infty}d\widetilde{\tau}\int_{-\infty}^{\infty}d\widetilde{\tau}'\,\frac{\chi(\widetilde{\tau})\chi(\widetilde{\tau}')}{\widetilde{\tau}-\widetilde{\tau}'}\left(\frac{f(\tau)-f(\tau')}{(\tau-\tau')\left(f'(\tau)\right)^{1/2}\left(f'(\tau')\right)^{1/2}}\right),$$

which gives as condition for the invariance

$$f(\tau)-f(\tau')=\left(\tau-\tau'\right)\left(f'(\tau)\right)^{1/2}\left(f'(\tau')\right)^{1/2}.$$

Taking the square of the two terms one recovers the relation which guarantees the invariance of the CL action, and which establishes that $f(\tau)$ is a transformation of $\mathrm{SL}(2,\mathbb{R})$.

When we consider the full superstring action containing both the bosonic and the fermionic terms there is a residual supersymmetry which survives the integration over the oscillator modes. The dissipative fermionic term (3.10) adds to the CL term (3.14) coming from the integration on the bosonic modes, producing a non-local supersymmetric dissipative term. After a trivial rescaling of the fields, it can be written as

$$S_{\mathrm{diss}}=\int dt_1\int dt_2\left(\frac{q(t_1)q(t_2)}{(t_1-t_2)^2}-\frac{\chi(t_1)\chi(t_2)}{t_1-t_2}\right), \tag{5.1}$$

which is invariant under the supersymmetry transformation

$$\delta q=\epsilon\chi\,,\qquad \delta\chi=\epsilon\frac{dq}{dt}\,, \tag{5.2}$$

where ϵ is a real constant anticommuting parameter. In fact, (5.1) can be easily written using superspace formalism. Let us introduce a Grassmann coordinate θ and the super-variable

$$X(t,\theta)=q(t)+\theta\chi(t)\,.$$

The superspace possesses the symmetry

$$\delta_\varepsilon t = \varepsilon t\,, \qquad \delta_\varepsilon \theta = \varepsilon\,, \qquad \varepsilon^2 = 0\,, \qquad \{\varepsilon, \theta\} = 0\,.$$

The supersymmetry generator is

$$Q = \partial_\theta - \theta \partial_t\,.$$

Then the action (5.1) can be obtained integrating over the θ's the following expression

$$S_{diss} = \int dt_2 dt_1 d\theta_2 d\theta_1 \frac{X(t_1,\theta_1) X(t_2,\theta_2)}{z_{12}} \tag{5.3}$$

where $z_{12} = t_1 - t_2 + \theta_1\theta_2$. Let us notice that there is an obvious generalization of the non-local supersymmetric action (5.3) i.e.

$$\begin{aligned} S &= \int dt_2 dt_1 d\theta_2 d\theta_1 X(t_1,\theta_1) W(z_{12}) X(t_2,\theta_2) \\ &= \int dt_1 \int dt_2 \left(q(t_1) W'(t_1 - t_2) q(t_2) - \chi(t_1) W(t_1 - t_2) \chi(t_2) \right)\,, \end{aligned} \tag{5.4}$$

with $W(t_1 - t_2) = -W(t_2 - t_1)$. In fact, another example of action of the form (5.4) is provided by the theory on the circle obtained by adding the action (3.8) of fermions periodic in time to the CL term (3.13).

Actually, one can check that the action (5.3) is invariant under the larger group of super-conformal transformations

$$\begin{aligned} t &\to \frac{at+b}{ct+d} + \frac{-\delta t + \varepsilon}{(ct+d)^2}\theta \\ \theta &\to \frac{\theta}{ct+d} + \frac{-\delta t + \varepsilon}{ct+d} \end{aligned} \tag{5.5}$$

where a, b, c, d are real constants and δ, ε are real anticommuting constants satisfying the relation $ad - bc = 1 - \delta\varepsilon$. These transformations are the real subgroup of the super-Moebius transformation acting on a complex commuting variable z and a couple $\theta, \overline{\theta}$ of Grassmann variables [14], (see also [15]). The corresponding Lie algebra contains three bosonic charges: the hamiltonian H, which is the generator of time translations, the time dilatation generator D, and K which generates conformal transformations. The remaining anticommuting charges are the generator of supersymmetry, Q, and the generator of special (conformal) transformations, S. Explicitly, following the notations of ref. [16], the algebra is characterized by the following non vanishing (anti-)commutation relations

$$\begin{aligned} &[H,D] = iH \qquad &&[H,K] = 2iD \qquad &&[D,K] = iK \\ &\{Q,Q\} = H \qquad &&[D,Q] = -\frac{i}{2}Q \qquad &&[K,Q] = \frac{i}{2}S \\ &\{S,S\} = K \qquad &&[D,S] = iS && \\ &\{Q,S\} = -D \qquad &&[H,S] = -iQ\,. && \end{aligned} \tag{5.6}$$

The same algebra (5.6), was considered in the study of the geometry of superconformal quantum mechanics made in ref. [17], where it was classified as the $\mathcal{N} = 1B$ extension of the Poincarè subalgebra to $Osp(2|1)$ superconformal algebra. The (super-)conformal invariance of the dissipative term makes it play a dominant role in determining the long range physical properties of a dissipative system.

6. Supersymmetric dissipative quantum mechanics and other boundary interactions

The non-local supersymmetric dissipative term can be added to the usual supersymmetric kinetic term [18], which is also invariant under (5.2)

$$S_{kin} = -\frac{1}{2}\int dt \int d\theta \left(D^2 X D X\right) = \frac{1}{2}\int dt \left(\left(\frac{dq}{dt}\right)^2 - \chi\frac{d\chi}{dt}\right) \tag{6.1}$$

where $D = \partial_\theta + \theta\partial_t$, and $\{Q, D\} = 0$. The system described by the total action $S_{kin} + S_{diss}$ is thus a supersymmetric extension of dissipative quantum mechanics. Notice however that the kinetic term breaks the $\mathrm{SL}(2,\mathbb{R})$ invariance of (5.3), and (super-)conformal invariance is lost (Actually, (6.1) still has a hidden superconformal invariance [19]). The physical interpretation of the model is better understood if we extend it to the case of more then one (super-)variable. Let us consider for instance the case of $D = 2k$ supervariables $X^\mu(t,\theta) = q^\mu(t) + \theta\chi^\mu(t)$, with $\mu = 1,\dots,D$. It is well known that the path integral on only the commuting variable q^μ gives the propagator of a scalar field in D dimension, while path integral on only the anticommuting variable χ^μ gives a pseudoclassical lagrangian representation of the spin (for simple discussions see for instance [20, 21]). This last fact is better understood when (6.1) is considered as the starting point of canonical quantization. As the kinetic term for the χ^μ in (6.1) is linear in time derivatives, half of the anticommuting variables are the conjugate momenta of the remaining half. Quantization leads then a to the irreducible $2^{D/2}$ spinor representation of the D-dimensional Grassmann algebra. (This is why we considered D even. A discussion about the partition function for a single anticommuting χ can be found in [22]). When computing the amplitude from (q^μ, χ^μ) to (q'^μ, χ^μ) (the quantity χ^μ remains constant due to the equation of motion $d\chi^\mu/dt = 0$), one obtains the propagator for a Dirac particle [18].

One can add another local supersymmetric term to introduce a potential into the action. Its form is familiar from recent studies on tachyon condensation in superstring theory [22, 23, 24].

$$S_T = \int dt \int d\theta \left(\Gamma D \Gamma + \Gamma T(X)\right) \tag{6.2}$$

where the scalar (super-)potential $T(X)$ represents in string theory the tachyon field and $\Gamma(t,\theta) = \eta(t) + \theta F(t)$ is a new anticommuting supervariable living on the string end-point (whose presence in string theory is needed in order to have correct space-time properties of the tachyon vertex operator). Again, one has to restrict to (NS) antiperiodic η, to avoid problems with zero-modes. Integration on θ, and functional integration on the auxiliary field F, lead to the following expression

$$S_T = -\frac{\alpha'}{8}\int dt \int dt' \left[\chi(t)\frac{\partial}{\partial q}T(q(t))\right] \mathrm{sign}(t-t') \left[\chi(t')\frac{\partial}{\partial q}T(q(t'))\right] - \frac{1}{4}\int dt\, T(q(t))^2$$

where we have restored α'. Notice that the scalar potential for q is positive definite.

Let us make some general remarks on the one-dimensional models obtained from an action containing (6.1), (5.3) and (6.2). Conformal invariance is lost in presence of (6.1) and (6.2) and one should then study the properties of the model under the renormalization group flow. In particular, one can hope that full conformal invariance is restored at some non trivial fixed point. The behavior of the various terms under scaling transformations is different. The non-local dissipative term is scaling invariant, while the kinetic term is an irrelevant operator and the term containing the tachyon potential is marginal or relevant. All these facts were well known in the case of the purely bosonic case [25, 26]. For instance, for what concerns the long range (IR) behavior of the model, the kinetic terms (which is an irrelevant operator) plays just the role of an UV cut-off and can be discarded in favor of simpler cut-off procedures in the course of renormalization analysis. It is then the dissipative term, which is scale-invariant, to play, in the quantum mechanical model, the role of inverse propagator, and so is the one-dimensional analogous of the laplacian operator in two dimensions. When the model contains more than one variable, one can add other interesting terms to the action. For instance, in the case of $D = 2$, one can couple the system to an external magnetic field obtaining models similar to the dissipative Hofstadter model, which has been widely studied for its remarkable duality properties [27, 28]. It would be interesting to study possible dualities of the supersymmetric version of that model.

7. Conclusions and outlook

With hindsight the derivation of the CL model from the dynamics of the open string end-point may seem not surprising, since, with its infinite number of oscillation modes, the open string just provides a model of the bath of oscillators required in the Caldeira-Leggett approach to dissipation. This point of view is advocated in [29], where it is stated that when the oscillator spectral weight vanishes linearly at low frequencies, i.e. in the case of Ohmic dissipation, the set of oscillators may be represented by a (1+1) dimensional quantum conformal field theory of free massless bosons living on a fictitious half-line. An alternative approach to dissipation can be found in [30, 31] and [32]. In those papers a reformulation of the fluctuation-dissipation theorem was introduced in such a manner that the basic idea of simulating the dissipative environment by an infinite number of bosonic oscillator was realized without manifestly introducing them. It is interesting that this formulation was also extended to fermions. In particular in ref. [32], these ideas where applied to a fermionic detector (a single quantum mechanical fermionic degree of freedom) coupled to a Dirac field which plays the role of a fermionic dissipative environment.[1]

Let us finally stress the main result of this paper. We have constructed new dynamical models extending the fermionic particle model described by the lagrangian (6.1): we have shown how to include dissipative effects by means of a straightforward extension of the CL term. Once again, string theory has proven to be a valuable tool to derive these results, following the seminal ideas of Callan and Thorlacius.

[1] The following Lorentz invariant non-local term for spinor fields in four-dimensional Minkowski spacetime

$$\frac{i\eta}{\pi} \int d^3x \int dtdt' \, \bar{\psi}(t, \mathbf{x}) \, \frac{1}{t-t'} \, \psi(t', \mathbf{x})$$

has been proposed for CPT violation for fermions (neutrinos) [33]. Criticisms to the physical interpretation of this model have been arisen [34]. We just notice that it reduces to our dissipative action if we consider fields independent of the spatial coordinates.

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